## **EXERCISES**

- 1. Write each fraction as an equivalent fraction with a denominator of 100. Then, write each fraction as a percent.
  - a.  $\frac{6}{10}$  b.  $\frac{13}{20}$  c.  $\frac{13}{25}$  d.  $\frac{3}{4}$  e.  $\frac{6}{5}$

2. Since 8 is not a divisor of 100, when we write the fraction  $\frac{3}{8}$  as a percent, the percent will have to have a tenths digit. One way to write  $\frac{3}{8}$  as a percent is to notice that  $\frac{3}{8}$  is half way between  $\frac{2}{8}$  and  $\frac{4}{8}$ , which means that the decimal equivalent of  $\frac{3}{8}$  is half way between 0.25 and 0.50, which is 0.375. Therefore, as a percent  $\frac{3}{8}$  is 37.5%. Use this strategy to write each of the following fractions as a percent. a.  $\frac{5}{8}$  b.  $\frac{14}{16}$  c.  $\frac{11}{40}$  d.  $\frac{12}{32}$  e.  $\frac{15}{24}$ 

- 3. All of the percents in question 2) can be expressed using only one decimal place. This is certainly not the case for every fraction. How might you express the fractions  $\frac{2}{3}$  and  $\frac{2}{7}$  as percents?
- 4. Looking at the fractions in the table, you will notice that each represents a percent that is reasonably small, which we can conclude by comparing the numerator and the denominator. Using what you already know about how fractions can be converted to percents, complete the table using mental math! The first row has been done for you as an example.

	Starting Fraction	$\frac{\boxed{}}{100}$	Decimal Value	Percent
a.	$\frac{1}{20}$	$\frac{5}{100}$	0.05	5% (not $50%$ )
b.	$\frac{3}{50}$			
C.	$\frac{2}{25}$			
d.	$\frac{3}{40}$			
e.	$\frac{6}{200}$			
f.	$\frac{36}{400}$			

	Starting Fraction	$\frac{\boxed{}}{100}$	Decimal Value	Percent
g.	$\frac{9}{360}$			

5. Consider the following set of shapes, which consist of rectangles and ovals. Notice that some shapes are small and some are tall.



- a. i. What percent of the shapes are small?
  - ii. What percent of the shapes are tall?
  - iii. Find the sum of the percents in part i) and part ii). Explain why this sum makes sense.
- b. i. What percent of the shapes are shaded?
  - ii. What percent of the shapes are not shaded?
  - iii. Find the sum of the percents in part i) and part ii). Explain why this sum makes sense.
- c. i. What percent of the shapes are small and shaded?
  - ii. What percent of the shapes are tall and not shaded?
  - iii. Find the sum of the percents in part i) and part ii). Explain why the sum is not equal to 100%.
- 6. Ahmad has just received the results of three assignments that he completed last week. His marks were
  - 34 out of 40 on his science fair project,
  - +  $\,64\,$  out of  $80\,$  on his geography presentation, and
  - 21 out of 25 on his French oral dictation.
  - a. Calculate each of Ahmad's marks as a percent.

- b. To summarize his results, Ahmad decides to calculate two average percentages:
  - First, he calculated the mean of the three percents in part a).
  - Second, he calculated the percent of total marks awarded out of total marks available for all three scores.

Explain whether these two percents should be the same. If not, which value should be higher. Then calculate both values and compare.

- 7. a. The side length of a regular hexagon is 2 cm. This side length is what percent of the perimeter of this hexagon? Round your answer to one decimal place.
  - b. A line is drawn connecting two diametrically opposite vertices of a regular hexagon of side length 2 cm.
    This line, along with three sides of the hexagon, form an isosceles trapezoid, as shown. This line is what percent of the perimeter of the trapezoid?
- 8. Rod wants to put down new laminate flooring in his family room. Each box of flooring will cover  $1.3 \text{ m}^2$ . Rod's room has an area of  $20 \text{ m}^2$ .



- a. What is the minimum number of boxes of flooring Rod needs to purchase?
- b. It is suggested that you buy 10% more flooring than what you need to account for waste from pieces that need to be cut. What is the suggested minimum number of boxes that Rod should purchase?
- 9. Each time Kim pours water from a jug into a glass, exactly 10% of the water remaining in the jug is used. What is the minimum number of times that she must pour the water into a glass so that less than half of the water remains in the jug?
- 10. The graph shows styles of music on a playlist. The playlist is currently composed of 35% Pop music and 65% Hip Hop music. Country music songs are added to the playlist so that now 40% of the songs are Country. If the ratio of Hip Hop songs to Pop songs remains the same, what percent of the total number of songs are now Hip Hop?



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**EXERCISES** 

1. Consider the following set of playing cards:



Source: Cards - Pavlo Stavnichuk/iStock/Getty Images

Identify each of the following statements as being true or false. If the statement is false, explain the mistake and then write the ratio that correctly describes the situation.

- a. The ratio of diamonds to hearts is  $2 \mbox{ to } 1.$
- b. The ratio of kings to all face cards is 2:2.
- c. The ratio of non face cards with an even number to non face cards with an odd number is 5 to 3.
- 2. Consider the following set of 10 cards containing pentagons, double daggers (‡), and music notes (  $\downarrow$  ).



The following ratios describe relationships that occur within the given set of cards. State 3 different relationships that each ratio could represent.

- a. 2 : 3
- b. 5:2
- c. 3:4

Team Name	Wins (W)	Losses (L)	Ties (T)
А	19	5	8
В	16	8	8
С	15	9	8
D	15	10	7
E	15	12	5
F	11	15	6
G	11	15	6
Н	10	13	9
I	10	13	9
J	9	18	5

3. The following table summarizes the statistics for all the matches from Major League Soccer.

- a. Identify the individual Wins to Losses to Ties ratios for teams B, F, and I.
   How many games have each of these three teams played?
- b. What fraction of games has team D won? What about team H?
- c. What fraction of games has team C lost? What about team J?
- d. What fraction of games has team A tied? What about team G?
- e. If each team has participated in the same number of games, the sum of numbers in each row will always equal a constant value. You should verify that this is true for the above table. What must be true about the row sums in the table if one league game was cancelled due to bad weather?
- 4. According to a certain recipe for stewed pears, you first need to prepare a mixture consisting of water and sugar and boil it for five minutes. To stew 4 pears, you combine 3 cups of water and 1 cup of brown sugar. If you plan to stew a total of 16 pears, how much water and brown sugar will you need to follow the recipe?
- 5. A chemistry teacher instructed her class to dilute vinegar by making a solution with a 2:3 ratio of vinegar to water. A student misheard her teacher and combined 40 mL of water and 60 mL of vinegar. How much water must she add to have the correct solution?

- 6. Ivan trained for a cross-country meet.
  - On Monday, he ran  $10 \ {\rm km}$ .
  - On Tuesday, he ran twice as far as he ran on Monday.
  - On Wednesday, he ran half as far as he ran on Tuesday.
  - On Thursday, he ran half as far as he ran on Wednesday.
  - On Friday, he ran twice as far as he ran on Thursday.

How far did Ivan run in total?

- 7. The perimeters of two squares have a ratio of 1 to 2. What is the ratio of the smaller square's area to the area of the larger square?
- 8. At a zoo, the ratio of animals from Asia to the animals from Africa is 2:1.
  - In the area of the zoo that contains animals from Asia, the ratio of birds to mammals is 5 : 2.
  - In the area of the zoo that contains animals from Africa, the ratio of birds to mammals is 1:3.

What is the ratio of all birds in the zoo to all mammals in the zoo?



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# **EXERCISES**

- 1. Write each of the following ratios in its simplest form. Following your conversion to simplest form, write what you did to convert the ratio to simplest form.
  - a.  $10:20 = \square:\square$ ; I divided both numbers in the ratio by  $\square$ .
  - b.  $28 \ {\rm to} \ 14$
  - c.  $48 \mbox{ to } 18$
  - d. 19:57
  - e. 96 : 21
  - f. 80 to 260
- Each of the following ratios represents the ratio of water to sugar used in mixing four pitchers of juice. How sweet the juice tastes will depend on the ratio of water to sugar. Order the ratios in order of increasing juice sweetness, starting with the ratio that produces the least sweet juice.

5:2 17:8 33:16 21:8

3. Two or more gears of different sizes engage to create what is called a gear train. The gears in a gear train are described by the number of teeth on each gear. Complete the following table containing gear train ratios.

Gear Train Teeth Ratio (number of teeth on each gear)	Gear Train Ratio (simplest form)
36:12	
96:16	
24 to $2$	
4 :	1:7
to 65	$1 \ { m to} \ 5$
25:10	

4. In Major League Baseball, all teams play the same number of games in the regular season.

- a. Last season, the team from Seattle won 78 games and lost 84 games. In simplest form, what was Seattle's win to loss ratio?
- b. In the same season, the major league baseball team playing in Atlanta has a win to loss ratio of 4:5. How many games did the team from Atlanta win?
- 5. A newly-constructed cinema in Millville is required to build a parking lot that contains at least two parking spaces for every ten seats in the new cinema. If the seating capacity of the new cinema is 360, use equivalent ratios to determine the minimum number of parking spaces that the new cinema must provide.
- 6. In a bin at the Gauss Grocery store, the ratio of the number of apples to the number of oranges is 1:4, and the ratio of the number of oranges to the number of lemons is 5:2. What is the ratio of the number of apples to the number of lemons?
- 7. A loonie is a \$1 coin and a dime is a \$0.10 coin. One loonie has the same mass as 4 dimes. A bag of dimes has the same mass as a bag of loonies. The coins in the bag of loonies are worth \$400 in total. How much are the coins in the bag of dimes worth?
- 8. A box contains a total of 400 tickets that come in three colours: blue, green and yellow. The ratio of blue to green tickets is 1:3. The ratio of green to yellow tickets is 1:4. What is the smallest number of tickets that must be drawn from the box to ensure that at least 50 tickets of one colour have been selected?

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## **EXERCISES**

- 1. A tennis court is approximately 24 m in length and 8 m in width. A soccer field is approximately 100 m in length and 75 m in width. Are the length and the width in the same proportion for each of these fields?
- 2. An original image has a length of 12 cm and width of 8 cm. The image is enlarged.
  - a. If the enlarged image has length 30 cm and width 15 cm, then explain how you know the enlarged image is **not** in proportion with the original.
  - b. If the enlarged image has a length of 30 cm and is in proportion with the original, then what is the width?
  - c. If the enlarged image has a width of 15 cm and is in proportion with the original, then what is the length?
- 3. Consider the following figures. In each figure, the polygon is divided into identical shapes.



Figure A







**Figure B** 





Which figures have the same proportion of shaded area to total area?

- 4. You are offered \$25 to make 12 beaded bracelets. The rate of payment is proportional to the number of bracelets that you make.
  - a. How much money would you be paid to make  $60 \mbox{ bracelets}?$
  - b. If you were paid \$175, then how many bracelets did you make?

5. While making the bracelets from question 4, you record the time it takes to make each batch of

12  bracelets.		
	Number of Bracelets	Time Spent (min)
	12	36
	24	60
	36	78
	48	90

- a. Is the number of bracelets that you can make proportional to the time spent? Explain your reasoning.
- b. Why might each batch of 12 bracelets not take the same amount of time to complete?
- 6. The side lengths of  $\Delta ABC$  are 3, 4, and 5 units.
  - a.  $\Delta ABC$  is enlarged, in proportion, and the largest side length of the enlarged triangle is 15 units. What are the lengths of the other two sides of this triangle?
  - b.  $\Delta ABC$  is reduced, in proportion, and the smallest side length of the reduced triangle is 0.75. What are the lengths of the other two sides of this triangle?
- 7. The length and width of two rectangles are proportional. This means that if the dimensions of the first rectangle are  $\ell$  and w and the dimensions of the second rectangle are L and W then the ratios  $\ell : w$  and L : W are equivalent.
  - a. If the length of the first rectangle and the width of the second rectangle are both 5 mm (that is,  $\ell = 5$  and W = 5), then give two possibilities for the remaining dimensions of the rectangles.
  - b. If the width of the first rectangle is 2 cm and the width of the second rectangle is 4 cm (that is, w = 2 and W = 4), then what is the ratio of the areas of the rectangles?

### **EXERCISES**

- 1. Write each of the following as a fraction and as a decimal.
  - a. 12.5%
  - b. 0.3%
  - c. 130%
  - d. 2000%
- 2. Write each fraction or decimal as a percent.
  - a.  $\frac{19}{40}$ b. 1.15 c. 10.55 d.  $\frac{15}{4}$ e. 0.004 f.  $4\frac{1}{5}$
- 3. Create a diagram to represent the quantity 32.5%.
- 4. One glass of pure orange juice can provide around 220% of the recommended daily intake of Vitamin C.
  - a. Write 220% as a fraction and as a decimal.
  - b. If the recommended daily intake of Vitamin C is 60 mg then approximately how many milligrams of Vitamin C are in one cup of pure orange juice?
- 5. Determine whether the given percentage is reasonable in the given context.
  - a. Rainfall this year was 120% of the average.
  - b. 105% of the bananas were eaten.
  - c. A company's annual profit increased by 0.5% from last year.

- d. A charity reached 200% of its donations goal for this month.
- 6. If the tax rate is 13%, then the total cost of an item is 113% of the ticket price.
  - a. If the ticket price of an item is \$75, estimate the total cost of the item by calculating 110% of the ticket price. Is your estimate too high or too low, and by approximately how much?
  - b. Calculate the total cost of the item from part a). How accurate was your estimate?
- 7. The following table contains data about the weekly sales of two shirt companies.

Week	Company A Sales	Company B Sales
1	15	240
2	36	360
3	108	648

- a. Calculate the increase in sales for each company from Week 1 to Week 2 and from Week 2 to Week 3.
- b. Calculate the percentage increase in sales for each company from Week 1 to Week 2 and from Week 2 to Week 3.
- c. From the information gathered in parts a) and b), which company is experiencing better growth in sales?
- 8. A sign displays information about travel times for a 2 kilometre dual purpose trail that is used by both bikers and pedestrians. The travel time to complete the trail for an average biker is 8 minutes and 24 minutes for an average pedestrian. You observe that these are accurate measurements of your travel times. On Monday you bike the trail and on Tuesday you decide to walk the trail.
  - a. What is the percentage increase in your travel time from Monday to Tuesday?
  - b. Assuming that you bike and walk at a constant speed, calculate the speeds at which you travel on Monday and Tuesday in km/min and km/h.
  - c. What is the percentage decrease in your speed from Monday to Tuesday? Does this answer depend on the units used in your speeds?